Name: Sofie Leung Date: _____

Pre Calculus 12 Section 2.4 Reciprocal Functions:

1. What does it mean to take the reciprocal of a number? Explain:

   flip denominator with numberator

   $2 \rightarrow \frac{1}{2}$

2. If the coordinate (a,b) is on the function $y = f(x)$, what will this coordinate become on the function $y = \frac{1}{f(x)}$?

   $(a, \frac{1}{b})$ $b \neq 0$

3. When you take the reciprocal of a function, what happens to the equation?

   $f(x) = \frac{1}{f(x)}$

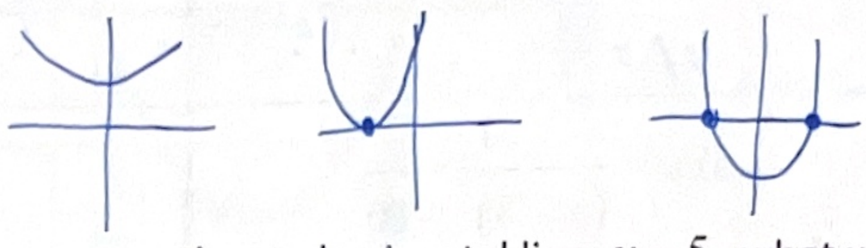
4. What is a vertical asymptote? How do you find the vertical asymptotes of a reciprocal function?

   A vertical line that the function can't cross

   They're the x-intercepts

5. If you taking the reciprocal function, how many vertical asymptotes can you have? Explain:

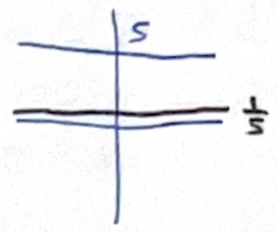
   0, 1, 2



6. Suppose you are given a horizontal line $y = 5$, what would the reciprocal of this function become?

   $(a, b) \rightarrow (a, \frac{1}{b})$

   $(x, \frac{1}{5})$



7. Suppose you are given a linear function, $y = mx + b$, how do you find the domain and range of the reciprocal of this function? Explain:

   R: $y \in R$, $y \neq 0$

   D: $x \in R$, $x \neq$ asymptotes

8. Suppose you are given a quadratic function, $y = x^2 - k$, how do you find the domain and range of the reciprocal of this function? Explain:

   D: $x \in R$, $x \neq \pm$ asymptotes

   R: $y > 0$, $y \leq \frac{1}{k}$ up

   R: $y < 0$, $y \geq \frac{1}{k}$ down

9. Suppose you are given the following coordinates on $y=f(x)$, what will these coordinates become on $y=\dfrac{1}{f(x)}$?

i $(5,-1)$ ii $(3,-2)$ iii $(10,20)$ iv $\left(6,-\frac{2}{3}\right)$ v $\left(\frac{4}{7},\frac{9}{4}\right)$ vi $(0,-1)$ vi $(-1,0.004)$

10. How do you find the invariant points of a reciprocal function? Explain:

$y = \pm 1$

11. Given each of the following reciprocal functions, Find the equation of the vertical asymptotes and the coordinates of ALL the invariant points?

a) $y=\dfrac{1}{5x-9}$

$y=1$: $0=5x-9$, $9=5x$, $x=\frac{9}{5}$

$y=-1$: $-1=5x-9$, $8=5x$, $x=\frac{8}{5}$

$VA=\frac{9}{5}$

$y=1$: $1=5x-9$, $5x=10$, $2=x$

$(2,1)$ $\left(\frac{8}{5},-1\right)$

b) $y=\dfrac{1}{\frac{2}{3}x-11}$

$y=1$: $1=\frac{2}{3}x-11$, $12=\frac{2}{3}x$, $x=18$

$y=-1$: $-1=\frac{2}{3}x-11$, $10=\frac{2}{3}x$, $x=15$

$0=\frac{2}{3}x-11$, $11=\frac{2}{3}x$, $x=\frac{33}{2}$

$VA=\frac{33}{2}$ $(18,1)$ $(15,-1)$

c) $y=\dfrac{1}{2x^2-9}$

$y=1$: $0=2x^2-9$, $9=2x^2$, $4.5=x^2$, $x=\pm\sqrt{\frac{9}{2}}$

$y=-1$: $1=2x^2-9$, $10=2x^2$, $5=x^2$, $x=\pm\sqrt{5}$

$VA=\pm\frac{3\sqrt{2}}{2}$ $(\pm\sqrt{5},1)$

$y=-1$: $-1=2x^2-9$, $8=2x^2$, $4=x^2$, $x=\pm 2$

$(\pm 2,-1)$

d) $y=\dfrac{1}{(x-3)^2-16}$

$y=1$: $0=(x-3)^2-16$, $16=(x-3)^2$, $\pm 4=x-3$, $x=7,-1$

$y=1$: $1=(x-3)^2-16$, $17=(x-3)^2$, $\pm\sqrt{17}=x-3$, $x=\pm\sqrt{17}-3$

$y=-1$: $-1=(x-3)^2-16$, $15=(x-3)^2$, $\pm\sqrt{15}=x-3$, $x=\pm\sqrt{15}-3$

$VA=7,-1$ $(\pm\sqrt{17}-3,1)$ $(\pm\sqrt{15}-3,-1)$

e) $y=\dfrac{1}{8x^2-10x-3}$

$0=8x^2-10x-3$, $x=\dfrac{10\pm\sqrt{10^2-4\cdot 8\cdot -3}}{16}$, $=\dfrac{10\pm 14}{16}$

$VA=\frac{3}{2},-\frac{1}{4}$

$y=1$: $1=8x^2-10x-3$, $0=8x^2-10x-4$, $x=\dfrac{10\pm\sqrt{10^2-4\cdot 8\cdot -4}}{16}$, $=\dfrac{5\pm\sqrt{57}}{8}$

$y=-1$: $-1=8x^2-10x-3$, $0=8x^2-10x-2$, $x=\dfrac{10\pm\sqrt{10^2-4\cdot 8\cdot -2}}{16}$, $=\dfrac{5\pm\sqrt{41}}{8}$

$\left(\frac{5\pm\sqrt{57}}{8},1\right)$ $\left(\frac{5\pm\sqrt{41}}{8},-1\right)$

f) $y=\dfrac{1}{2x^2-7x-15}$

$0=2x^2-7x-15$, $x=\dfrac{7\pm\sqrt{7^2-4(-15)\cdot 2}}{4}$, $=\dfrac{7\pm 13}{4}$, $=5,-\frac{3}{2}$

$VA=5,-\frac{3}{2}$

$y=1$: $1=2x^2-7x-15$, $0=2x^2-7x-16$, $x=\dfrac{7\pm\sqrt{7^2-4(-16)\cdot 2}}{4}$, $=\dfrac{7\pm\sqrt{177}}{4}$

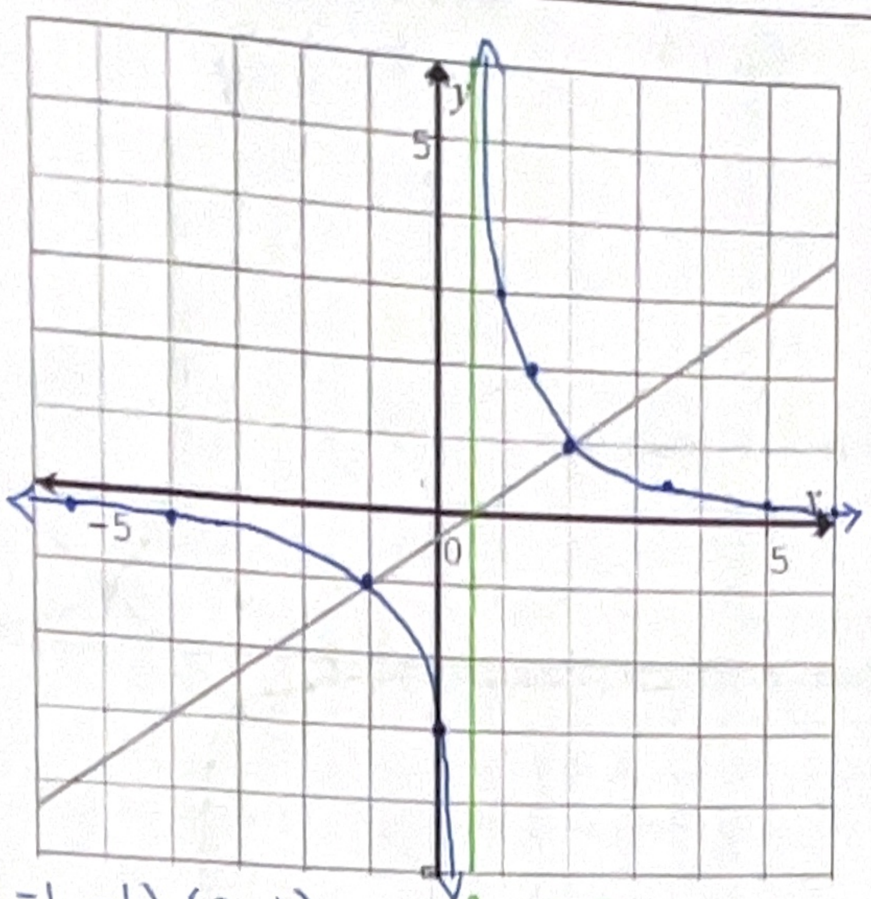
$\left(\frac{7\pm\sqrt{177}}{4},1\right)$

$y=-1$: $-1=2x^2-7x-15$, $0=2x^2-7x-14$, $x=\dfrac{7\pm\sqrt{7^2-4(2)(-14)}}{4}$, $=\dfrac{7\pm\sqrt{161}}{4}$

$\left(\frac{7\pm\sqrt{161}}{4},-1\right)$

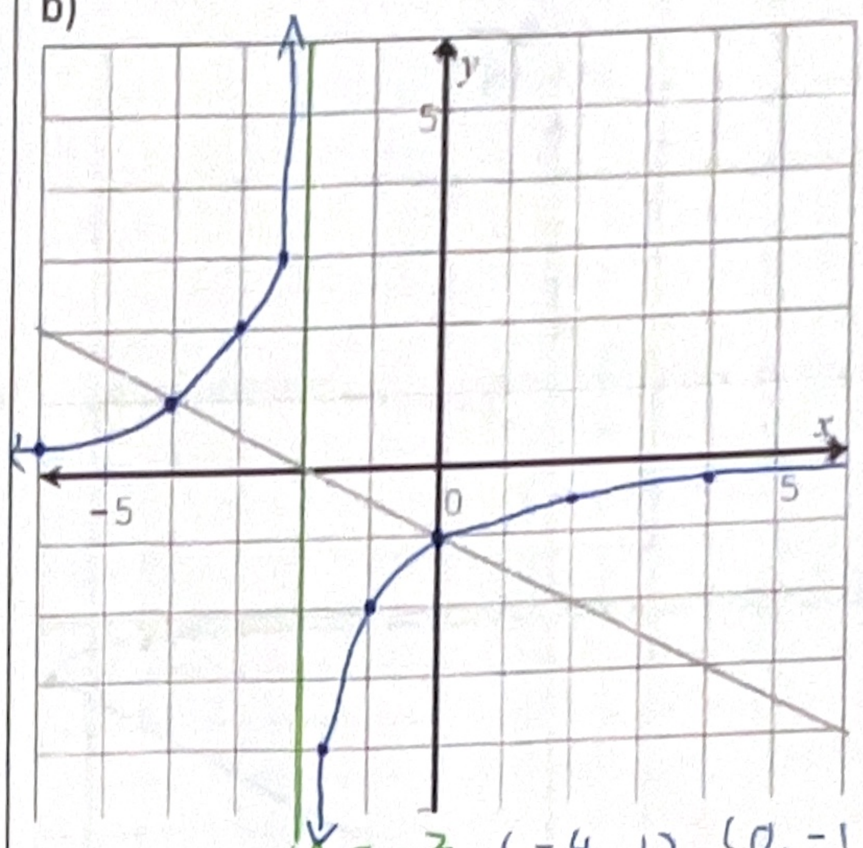
12. Given the graph of $y = f(x)$, graph the reciprocal function $y = \frac{1}{f(x)}$ on the same graph. Label the vertical asymptotes and the invariant points on the graph:

a)



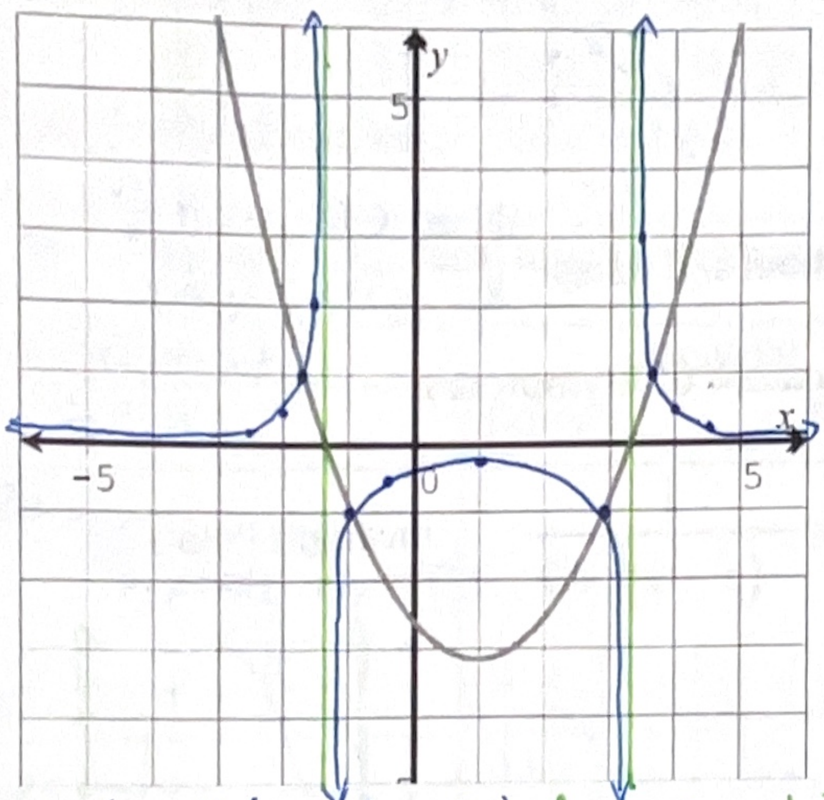
(-1,-1) (2,1) VA = 0,5

b)



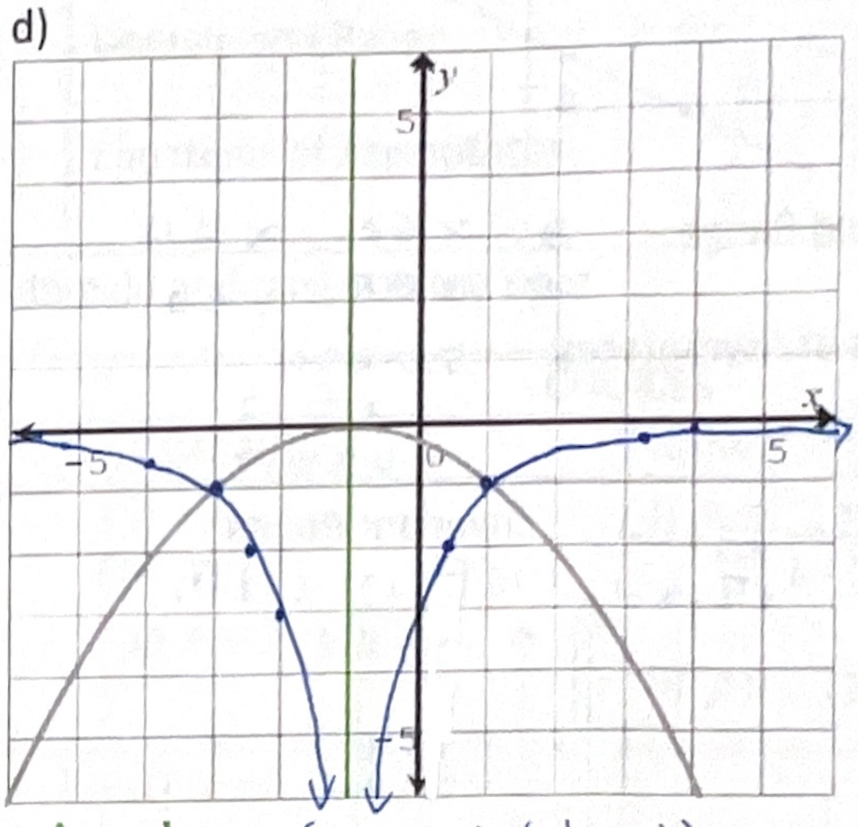
VA = -2 (-4,1) (0,-1)

c)



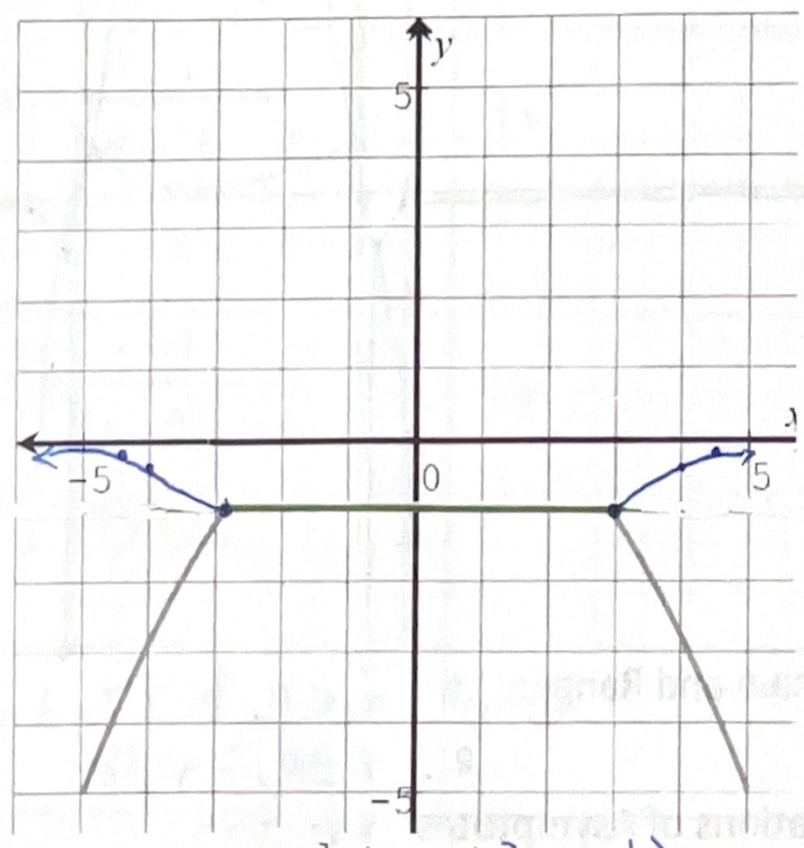
(3,-1) (-1,-1) (3,7,1) (-1·7,1) VA = 3,3 , -1,3

d)



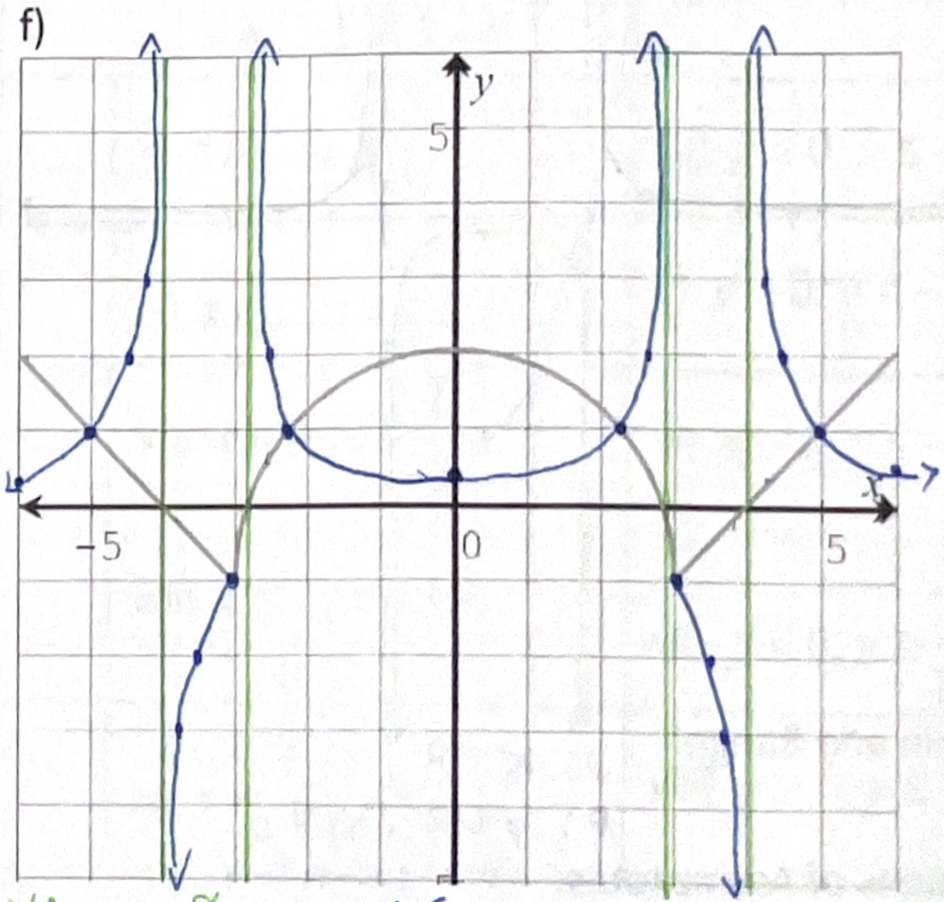
VA = -1 (-3,-1) (1,-1)

e)



$(-3 \leq x < 3, -1)$

f)



VA = -2,8 , 2,8 (-5,1) (-3,-1) (2,3,1) (-2,3,1) (3,-1) (5,1)

-4, 4

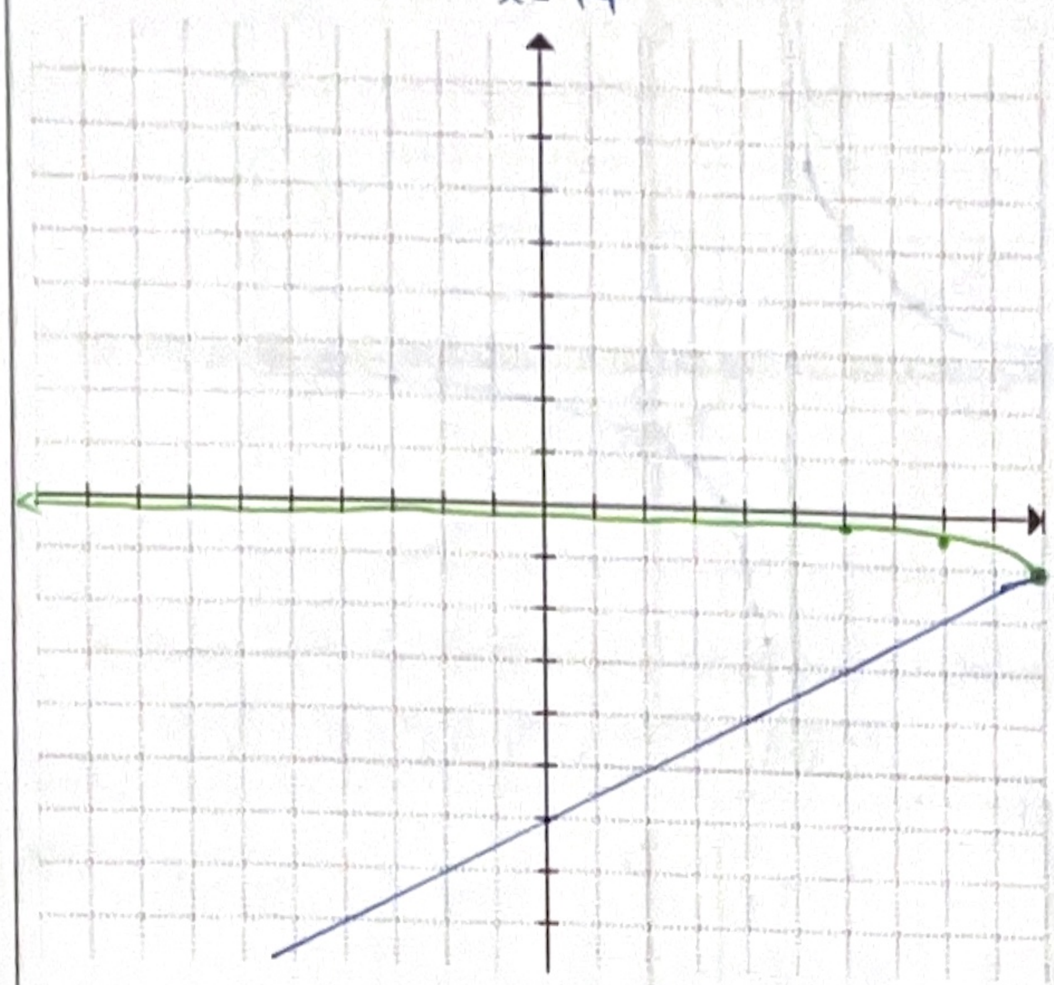
13. Graph the each of the following reciprocal functions. Indicate the asymptotes, coordinates of the invariant points, domain and range:

$y = \dfrac{1}{\frac{1}{2}x - 6}$

Invariant Points: (10;1) (14,1)

$1 = \frac{1}{2}x - 6$

$x = 14$



Domain and Range:

D: $x \in R$, $x \neq 12$

R: $y \in R$, $y \neq 0$

Equations of Asymptotes: $\frac{1}{2}x - 6 = 0$

$x = 12$

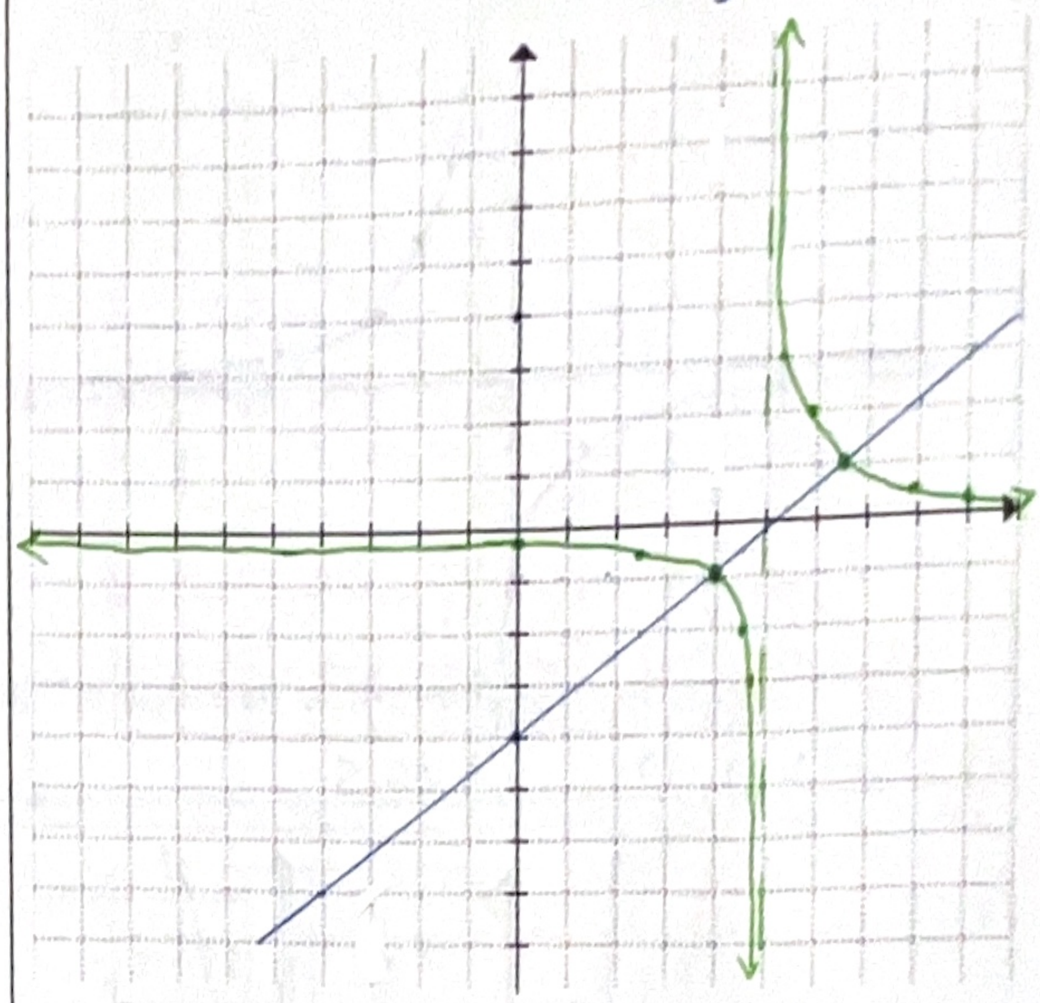
$VA = 12$

$y = -\dfrac{1}{\frac{3}{4}x + 4}$

Invariant Points (4, -1) $(\frac{20}{3}, 1)$

$-\frac{3}{4}x + 4 = -1$

$x = \frac{20}{3}$



Domain and Range:

D: $x \in R$, $x \neq \frac{16}{3}$

R: $y \in R$, $y \neq 0$

Equations of Asymptotes $-(-\frac{3}{4}x + 4) = 0$

$x = \frac{16}{3}$

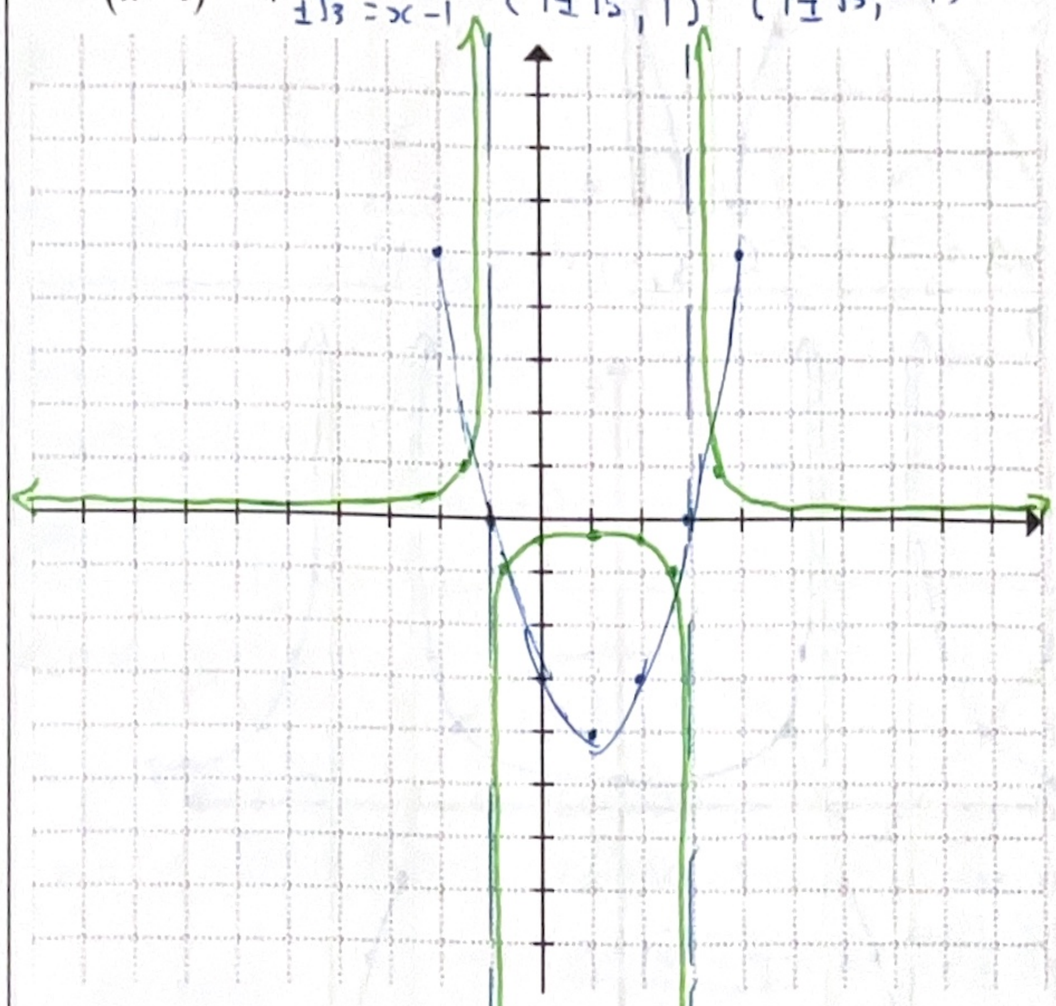
$y = \dfrac{1}{(x-1)^2 - 4}$

$\pm\sqrt{5} = x - 1$

$\pm\sqrt{3} = x - 1$

Invariant Points

$(1 \pm \sqrt{5}, 1)$ $(1 \pm \sqrt{3}, -1)$



Domain and Range:

D: $x \in R$, $x \neq 3, -1$

R: $y > 0$, $y \leq -\frac{1}{4}$

Equations of Asymptotes: $0 = (x-1)^2 - 4$

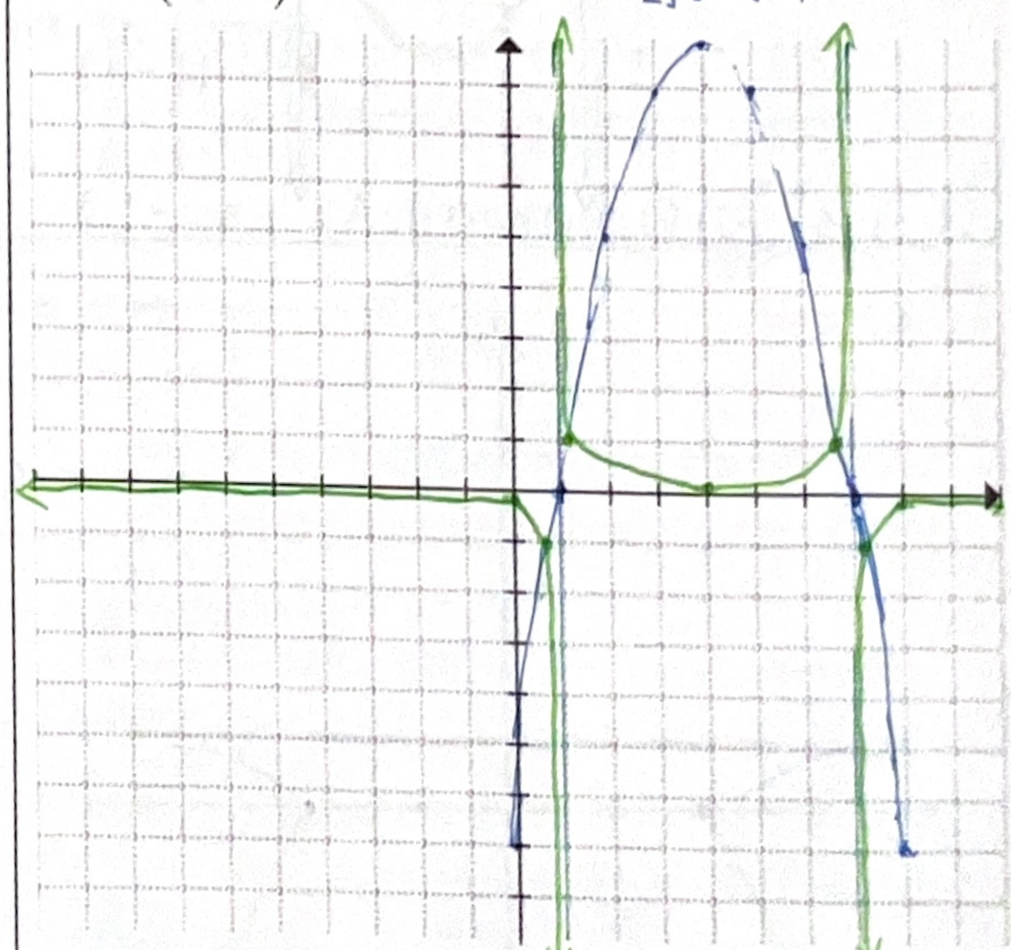
$\pm 2 = x - 1$

$x = 3, -1$

$y = \dfrac{1}{-(x-4)^2 + 9}$

Invariant Points $(4 \pm \sqrt{8}, 1)$ $(4 \pm \sqrt{10}, -1)$

$\pm\sqrt{8} = x - 4$ $\pm\sqrt{10} = x - 4$



Domain and Range:

D: $x \in R$, $x \neq 7, 1$

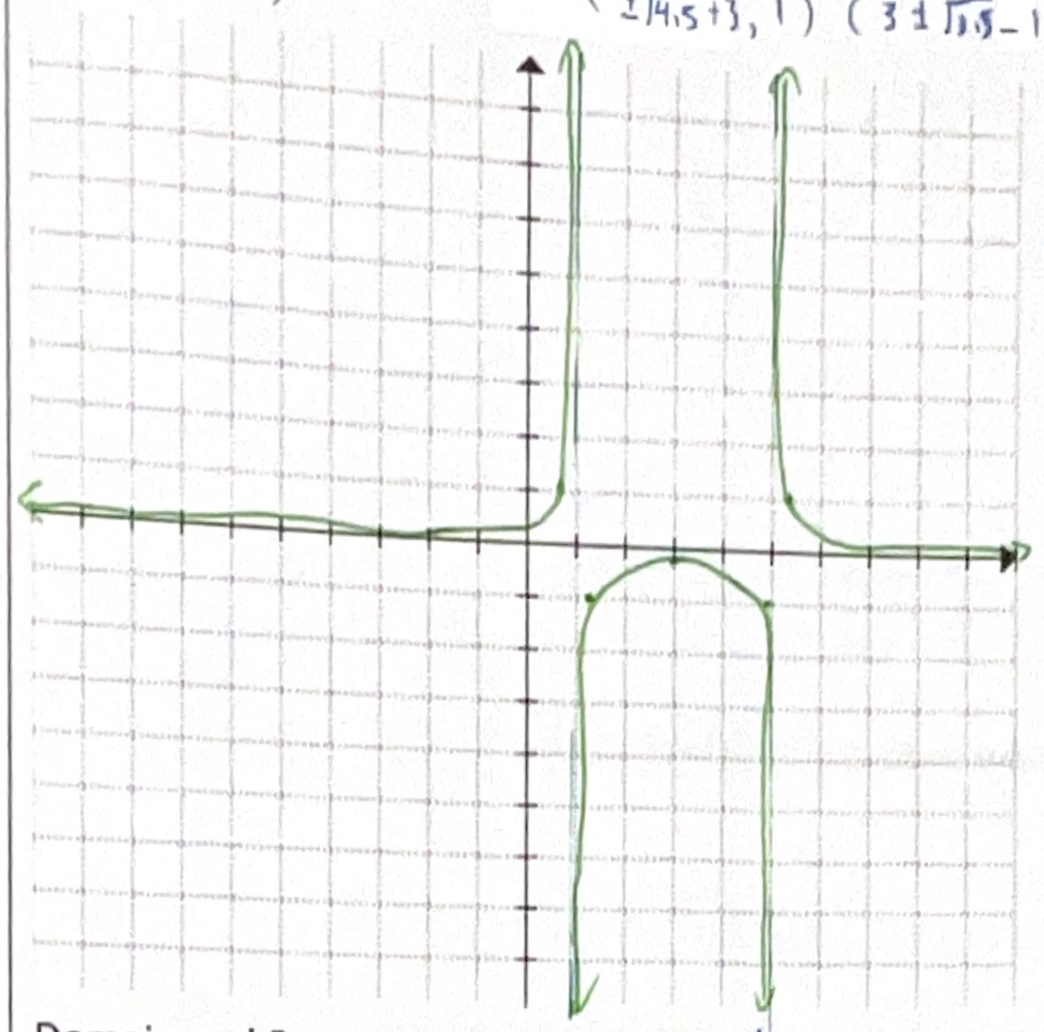
R: $y \geq \frac{1}{9}$, $y < 0$.

Equations of Asymptotes $\pm 3 = x - 4$

$x = 7, 1$

$$y = \frac{1}{2(x-3)^2 - 8}$$

Invariant Points: $(\pm\sqrt{4.5}+3, 1)$ $(3 \pm \sqrt{3.5}, -1)$



Domain and Range: D: $x \in R, x \neq 5, 1$

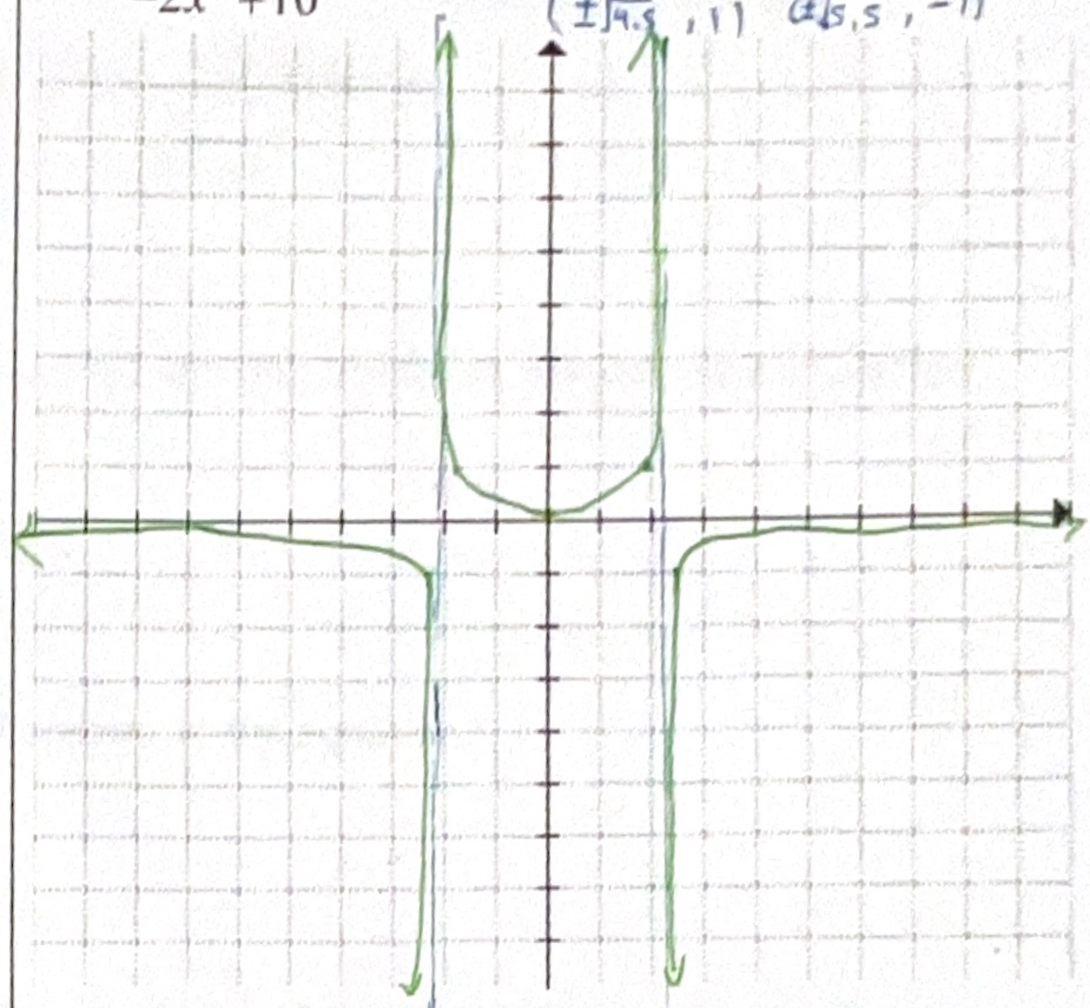
R: $y > 0, y \leq -\frac{1}{8}$

Equations of Asymptotes: $x = -3 \pm 2$

$= 5, 1$

$$y = \frac{1}{-2x^2 + 10}$$

Invariant Points: $(\pm\sqrt{4.5}, 1)$ $(\pm\sqrt{5.5}, -1)$



Domain and Range: D: $x \in R, x \neq \pm\sqrt{5}$

R: $y \geq \frac{1}{10}, y < 0$

Equations of Asymptotes: $\pm\sqrt{5}$

14. Match each reciprocal function with the correct domain and range on the right:

Functions:	ANSWER		CHOICES	
	Domain Range:		DOMAINS	RANGES:
a) $y = \frac{1}{3x-5}$ $x \neq \frac{5}{3}$	i		i) $x \in \square$; ii) $x \in \square; x \neq 2$	I) $y \in \square; y \neq 0$; II) $y \in \square; y \neq \frac{5}{3}$
b) $y = \frac{1}{2x-4}$ $x = 2$	ii i		iii) $x \in \square; x \neq 1,5$; iv) $x \in \square; x \neq 5,-1$	III) $y \in \square$
c) $y = \frac{1}{(x-2)^2 - 9}$ $\pm 3 = x-2$ $\frac{1}{9}$	iv iv		v) $x \in \square; x \neq \frac{3}{5}$; vi) $x \in \square; x \neq -1,-7$	IV) $y > 0, y \leq -\frac{1}{9}$; v) $y > 0, y < -\frac{1}{9}$
d) $y = \frac{1}{(x-3)^2 - 4}$ $x-3 = \pm\sqrt{2}$ $\frac{1}{4}$	ix vi		Vii) $x \in \square; x \neq \frac{1}{2}$; viii) $x \in \square; x \neq 1,7$	vi) $y > 0, y \leq -\frac{1}{4}$; vii) $y < 0, y \geq \frac{1}{18}$
e) $y = \frac{1}{-2(x-4)^2 + 18}$ $\pm 3 = x-4$ $\frac{1}{18}$	viii vii		ix) $x \in \square; x \neq 1,5$	viii) $y > 0, y \leq \frac{1}{9}$